Lecture 3 - Lasers

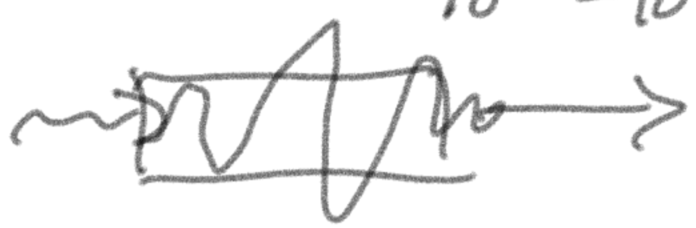
$$\frac{dN_2}{dt} = -A_{21}N_2 - \underbrace{\frac{A_{21}\lambda^2}{8\pi n^2} g(\nu)}_{\text{cross-section}} \frac{I}{h\nu}\left(N_2 - \frac{g_2}{g_1}N_1\right)$$

$\frac{I}{h\nu}$ → # of photons

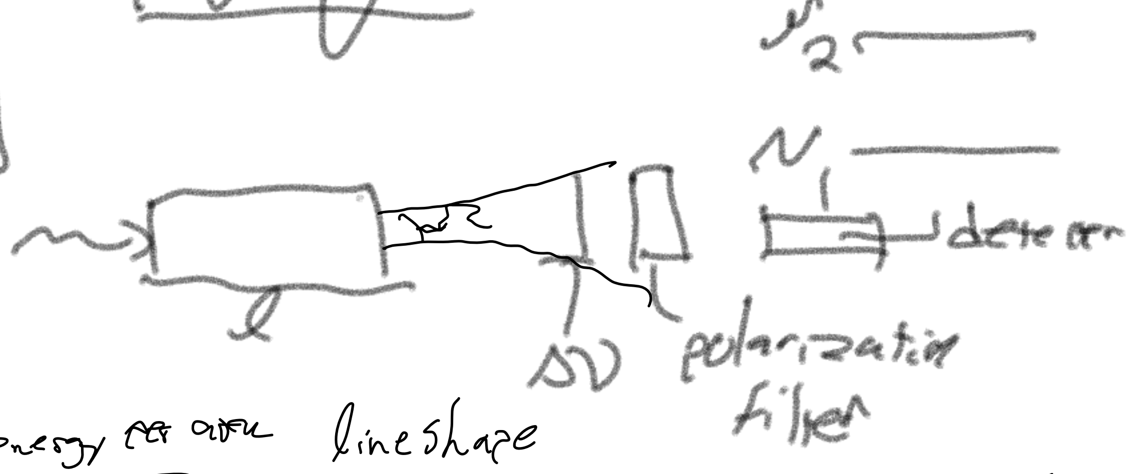
cross-section [cm^2]

$10^{-14} - 10^{-21}$

$$\frac{dI}{dz}$$



$I\left[\frac{W}{cm^2}\right]$



$\rho = \frac{I}{c/n}$

ΔI = energy per area, lineshape

	1	2	3	4	5	6
	energy	rate per atom	lineshape	prob. correct polarization	prob. solid angle	# of atoms interacting
stimulated	$+h\nu$	$\times B_{21}\frac{I_\nu}{c/n}$	$\times g(\nu)$	$\times 1$	$\times 1$	$\times N_2\Delta z$
absorption	$-h\nu$	$\times B_{12}\frac{I_\nu}{c/n}$	$\times g(\nu)$	1	$\times 1$	$\times N_1\Delta z$
spont. emission	$+h\nu$	$\times A_{21}\Delta\nu$	$\times g(\nu)$	$\times \frac{1}{2}$	$\times \frac{d\Omega}{4\pi}$	$\times N_2\Delta z$

$$\frac{dI}{dz} = -\left[\frac{h\nu}{c/n}\left(B_{21}N_2 - B_{12}N_1\right) g(\nu)\right] I_\nu$$

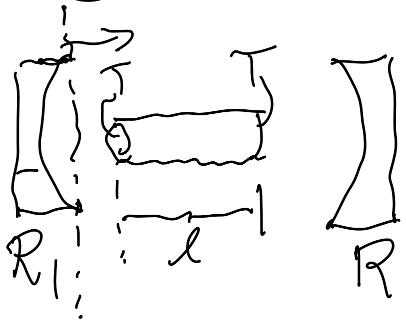
$$+ \frac{1}{2}\left[h\nu\, A_{21} N_2\, g(\nu)\,\Delta\nu \frac{d\Omega}{4\pi}\right] \to 0$$

$$\boxed{\frac{dI}{dz} = \left(\frac{A_{21}\lambda^2}{8\pi n^2} g(\nu)\right)\left(N_2 - \frac{g_2}{g_1}N_1\right) I_0}$$

σ

$$I(z) = A_0\, e^{\sigma\left(N_2 - \frac{g_2}{g_1}N_1\right) z}$$

$$I(z) = A_0\, e^{\sigma \Delta N z}$$



$$I_1 \to T e^{\sigma \Delta N l} \to R_2 T^2 e^{\sigma \Delta N l}$$

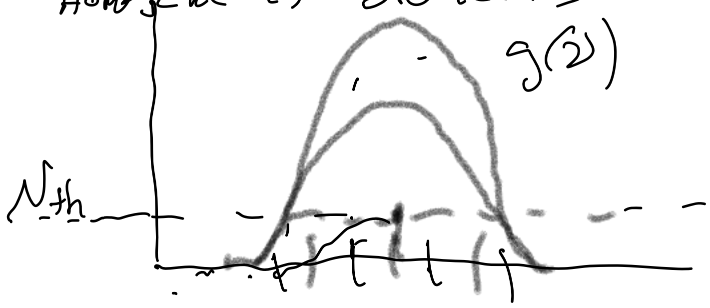
$$R_1 R_2 T^4 e^{2\sigma \Delta N l} = 1$$

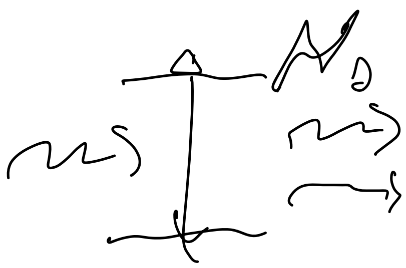
gain = loss

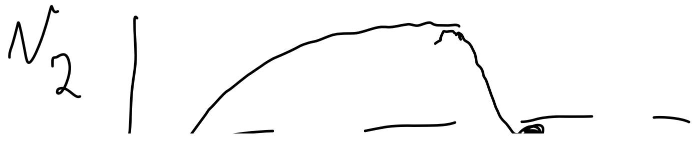
$$N_{th} \approx \frac{1}{2\sigma l} \ln\left(\frac{1}{R_1 R_2 T^4}\right)$$

threshold condition

Homogeneous broadening







pumping rate

$$A_{21} = \frac{1}{\tau_2}$$

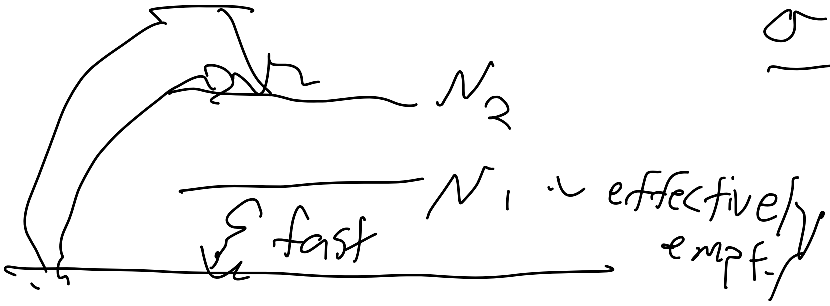
Steady State

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_2} - N_2 \frac{\sigma I}{h\nu} = 0$$

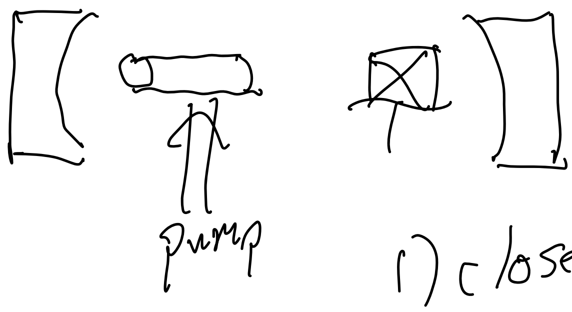
$$N_2 = \frac{R\tau_2}{1 + \frac{\sigma \tau_2}{h\nu} I} \qquad N_2 \longrightarrow N_{th}$$

$$\frac{h\nu}{\sigma \tau_2} = I_{sat}$$

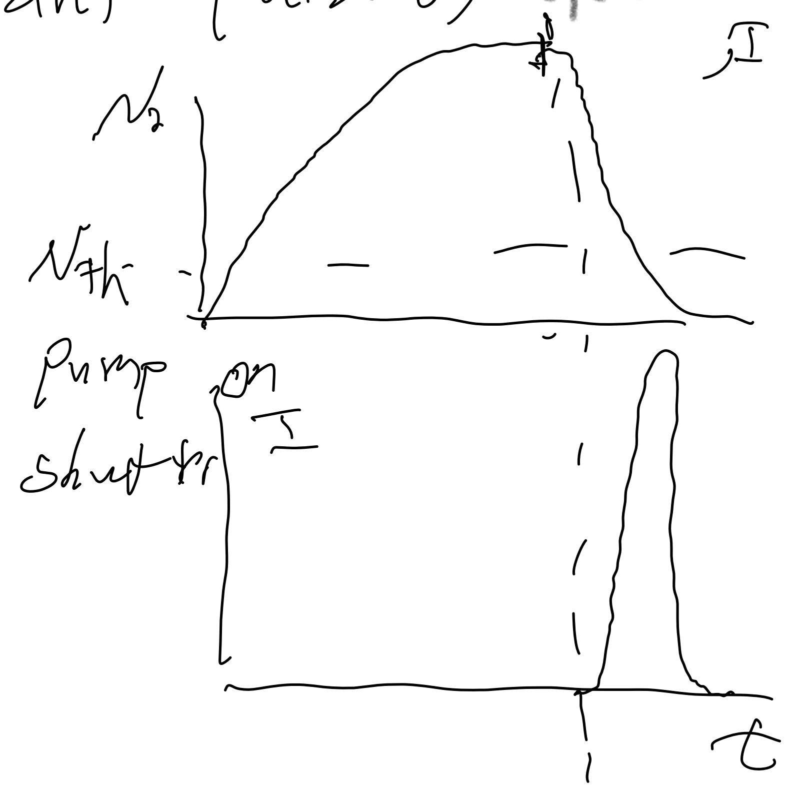
$$\frac{dI}{dz} = \sigma I (N_2 - N_1) \approx \sigma N_2 I = \frac{\sigma N \cdot I}{1 + I/I_{sat}}$$



Q-switching (Giant pulses)



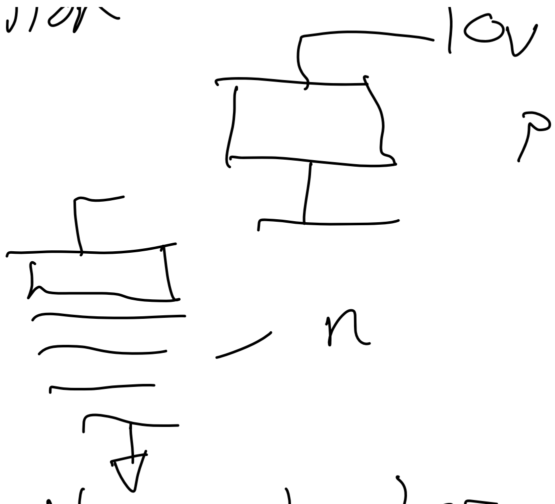
1) closed, pump on
2) open shutter

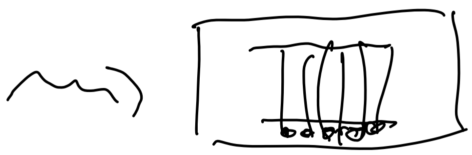


Pockels cell

rotates polarization

Acoustic-optic
PET

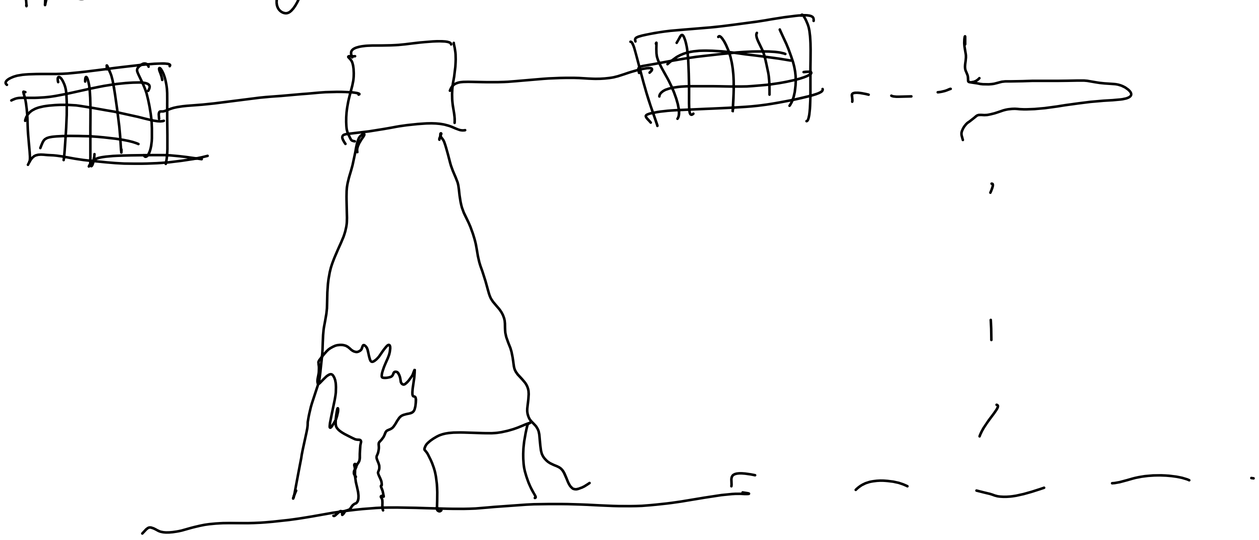




saturable absorber

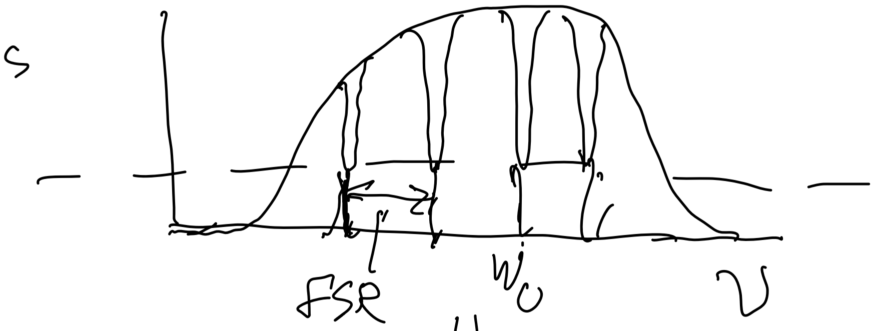
Giant pulses
~1 KHz, ~10 KHz

drilling
LIDAR - light detection and ranging

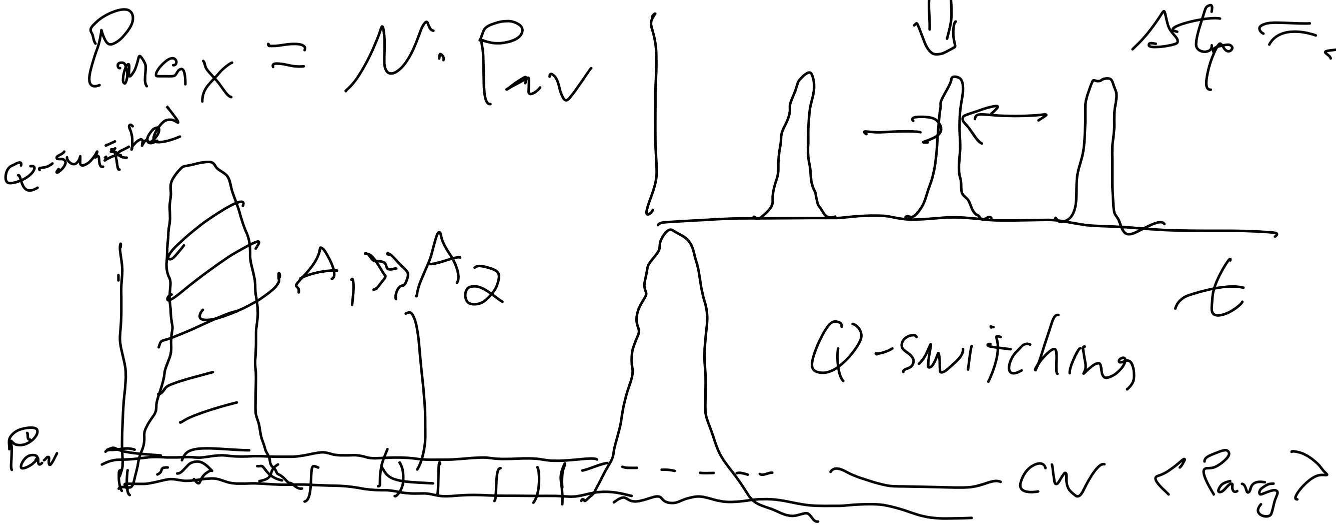


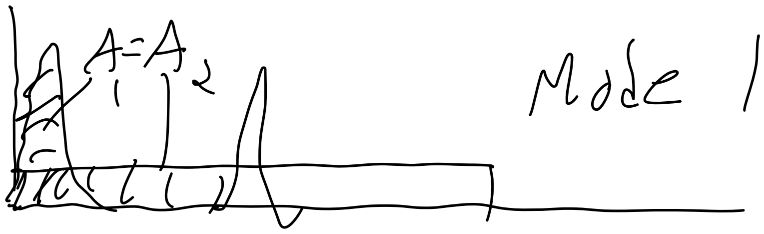
Mode locking

Inhomogeneous



$P_{max} = N \cdot P_{av}$ $\qquad \Delta t_p = \frac{\tau_{RT}}{N}$





Mode locking

$$e(t) = \sum_{add} E_n(t) \cdot e^{j(\omega_0 + n\omega_c)t + j\phi_n(t)}$$

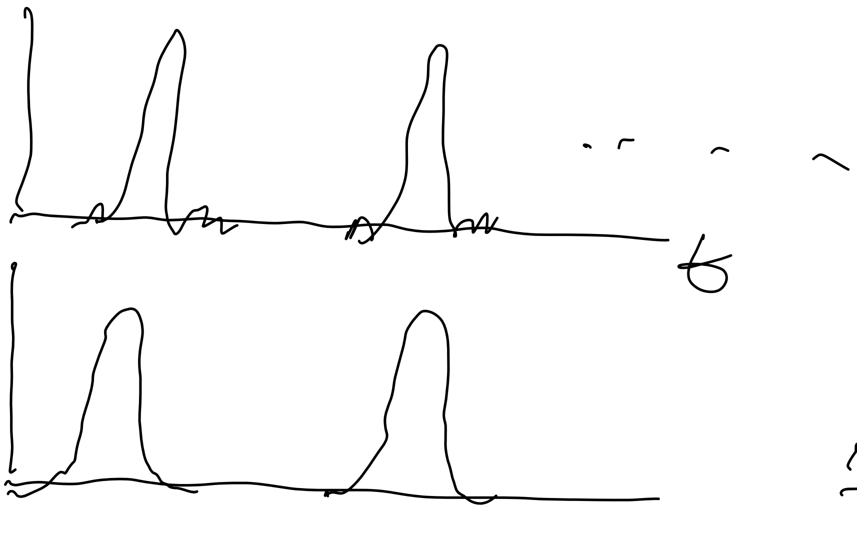
$$e(t) = E_0 e^{j\omega_0 t} \sum_{-\frac{N-1}{2}}^{\frac{N-1}{2}} e^{jn\omega_c t}$$

$$= E_0 e^{j\omega_0 t} \frac{\sin \frac{N}{2}\omega_c t}{\sin\left(\frac{\omega_c t}{2}\right)}$$

$$P = \frac{e(t)\, e^*(t)}{2\eta} = E_0^2 \operatorname{sinc}^2\left(\frac{N}{2}\omega_0 t\right)$$

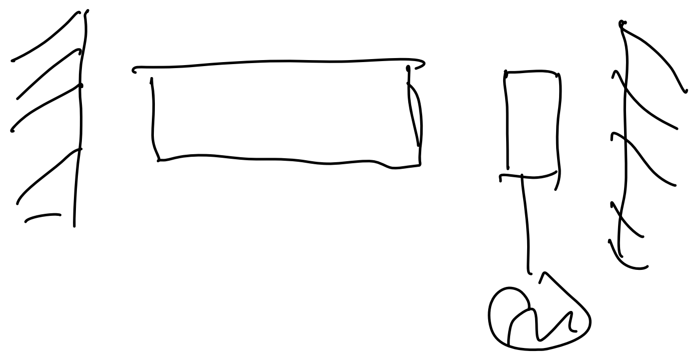
$\propto 1/t$

$$P_{max} = \frac{(NE_0)^2}{2\eta_0}$$

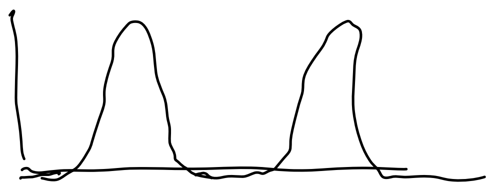


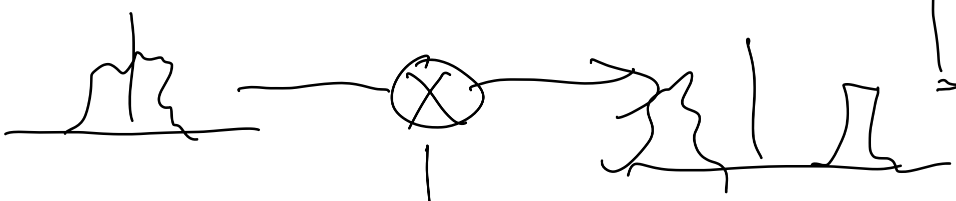
AM modulation

$$t_m(t) = e^{-\delta^2 \sin^2\left(\frac{\omega_m t}{2}\right)}$$



$\sin x \approx x$





(see handout)

Mode-Locking: Self-Consistency Approach (AM)[1]

Gain medium transfer function $T_g(\omega)$

$$\frac{\gamma(\omega)}{|\gamma(\omega_0)|} = \frac{1}{1 + j\left(\dfrac{\omega-\omega_0}{\Delta\omega/2}\right)} \approx 1 - j\left(\frac{\omega-\omega_0}{\Delta\omega/2}\right) - \left(\frac{\omega-\omega_0}{\Delta\omega/2}\right)^2$$

Starting pulse $e_1(t)$ and $E_1(\omega)$

$$e_1(t) = A\,e^{-at^2}\,e^{j\omega_0 t} = A\exp\!\left[-2\ln 2\left(\tfrac{t}{\Delta t_p}\right)^2\right]e^{j\omega_0 t}$$

$$E_1(\omega) = A\left(\frac{\pi}{a}\right)^{1/2}\exp\left[-\frac{(\omega-\omega_0)^2}{4a}\right]$$

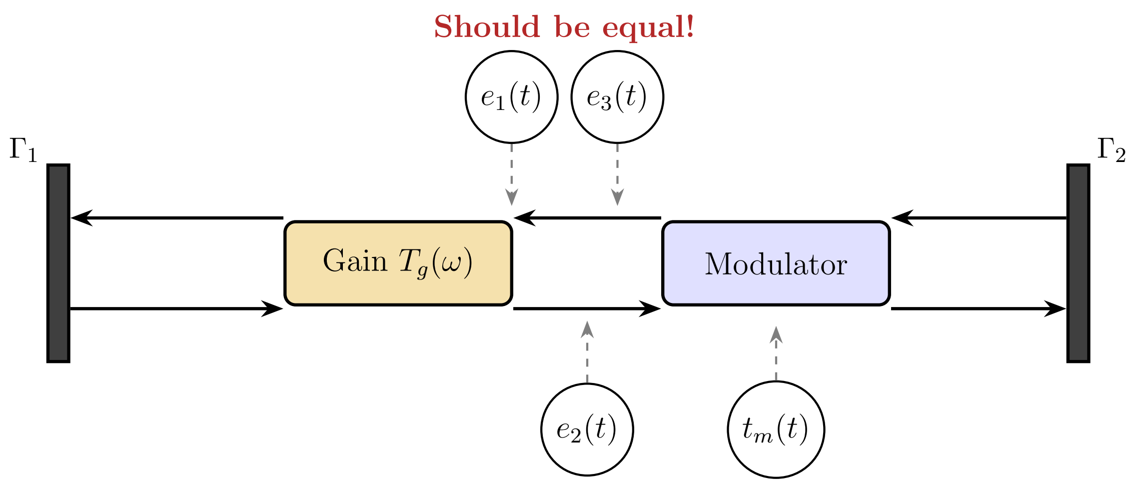
Self-consistency: $e_3 = e_1$

Mode-locking condition:

$$\boxed{\omega_m\,\tau_{\mathrm{RT}} = 2\pi m}$$

Steady-state pulse width:

$$\Delta t_p \propto \left(g_0\,\delta^2\,\omega_m^2\,\Delta\omega_g^2\right)^{-1/4}$$



After gain ×2 + Γ_1: $e_2(t)$, $E_2(\omega)$

$$e_2(t) = \frac{\Gamma_1 A}{2\sqrt{qa}}\,e^{g_0}\exp\left(-\frac{t^2}{4q}\right)e^{j\omega_0 t}$$

$$E_2(\omega) = \Gamma_1\,T_g^2(\omega)\,E_1(\omega), \quad q = \frac{1}{4a} + \frac{g_0}{(\Delta\omega/2)^2}$$

Amplitude modulator $t_m(t)$

$$t_m(t) = \exp\left[-\delta^2\sin^2\left(\frac{\omega_m t}{2}\right)\right] \approx \exp\left[-\frac{\delta^2\omega_m^2}{4}\,t^2\right]$$

After mod ×2 + Γ_2: $e_3(t)$

$$e_3(t) = \Gamma_2\,t_m^2(t)\cdot e_2(t) \propto \exp\left[-\left(\frac{1}{4q} + \frac{\delta^2\omega_m^2}{2}\right)t^2\right]e^{j\omega_0 t}$$

Symbol Definitions

ω_0	carrier (optical) frequency	ω_m	modulator drive frequency	τ_{RT}	cavity round-trip time
$\Delta\omega$	gain bandwidth (FWHM)	g_0	small-signal power gain coefficient	$\Gamma_{1,2}$	mirror field reflectivities
δ	modulation depth (AM index)	a	Gaussian pulse shape parameter; $a = 2\ln 2/\Delta t_p^2$	Δt_p	pulse duration (FWHM)
q	combined broadening parameter; $q = \frac{1}{4a} + \frac{g_0}{(\Delta\omega/2)^2}$	A	pulse amplitude	m	harmonic order ($m = 1$: fundamental ML)
$T_g(\omega)$	gain medium transfer function (freq. domain)	$t_m(t)$	modulator transmission (time domain)	e_n, E_n	field envelopes at labelled cavity points

[1] Based on lecture by Debdeep Jena (Cornell ECE4300 2016).